

New Results in Soft Gluon Physics

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We examine soft gluon physics, focusing on recently developed path integral methods. Two example applications of this technique are presented, namely the classification of soft gluon amplitudes beyond the eikonal approximation, and the structure of multiparton webs. The latter reveal new mathematical structures in the exponents of scattering amplitudes.

1 Introduction

It is well-known that QCD radiation leads to unstable results in perturbation theory when the momentum of the emitted radiation becomes low (“soft”). Typically, if ξ is some dimensionless energy variable representing the total energy carried by soft gluons, then one finds differential cross-sections of the form

$$\frac{d\sigma}{d\xi} = \sum_{n,m} \alpha_S^n \left[c_{nm}^0 \frac{\log^m(\xi)}{\xi} + c_{nm}^1 \log^m(\xi) + \dots \right] \quad (1)$$

involving large logarithms (of soft origin) to all orders in perturbation theory. Here the first set of terms can be obtained from the so-called *eikonal approximation*, in which the momentum of the emitted gluons formally goes to zero. Much is known already about these logarithms. The second set of terms arises from the *next-to-eikonal (NE) limit*, corresponding to a first order expansion in the momentum of the emitted gluons. These logarithms, although suppressed by a power of the energy scale ξ , can be numerically significant in many scattering processes.

In the soft phase-space region in which $\xi \rightarrow 0$, the above perturbation expansion breaks down in that all terms become large. The solution to this problem is to work out the logarithms to all orders in the coupling constant and sum them up (“resummation”). This is by now a highly developed subject, and many different approaches already exist for summing eikonal logarithms (e.g. Feynman diagram approaches, SCET). Here I will explain the basic idea using the *web* approach^{1,2,3}, and using the schematic scattering process shown in figure 1. This consists of a

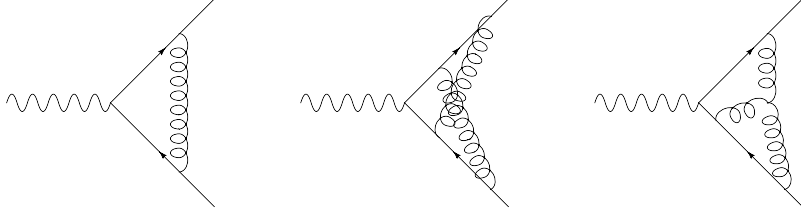


Figure 1: Schematic hard scattering process dressed by soft gluons.

hard interaction (in this case a virtual photon and quark pair of nonzero momentum), which is dressed by gluons (we do not distinguish real and virtual emissions in the figure). When these gluons become soft, this generates the large logs in eq. (1). However, one may show that the soft gluon diagrams *exponentiate*. That is, if \mathcal{A} is the amplitude for the Born interaction \mathcal{A}_0 dressed by any number of soft gluons, one has^{1,2,3}

$$\mathcal{A} = \mathcal{A}_0 \exp \left[\sum \tilde{C}_W W \right], \quad (2)$$

where the sum in the exponent is over soft-gluon diagrams W . This is a powerful result for two reasons. Firstly, large logs coming from the soft gluon diagrams sit in an exponent, thus get summed up to all orders in perturbation theory. Secondly, not all soft gluon diagrams have to be calculated. It turns out that only those which are irreducible (“webs”) need to be considered, and the first few examples are shown in figure 1. The webs have modified colour factors $\tilde{C}(W)$, which are not the usual colour factors of perturbation theory, and these are zero for non-webs. We see that crucial to resummation is the notion of exponentiation, and indeed this is common to all other approaches.

Having very briefly reviewed soft gluon physics, let us now focus on the following open problems:

1. *Can we systematically classify next-to-eikonal logarithms?* As remarked above, much less is known about the second set of logs in eq. (1) than the first set. A number of groups have looked at this in recent years^{4,5,6,7,8,9}.
2. *What is the equivalent of webs for multiparton processes?* The webs of^{1,2,3} are only set up for cases in which two coloured particles interact e.g. Drell-Yan production, deep inelastic scattering, $e^+e^- \rightarrow q\bar{q}$ etc. Recent work has tried to generalise the web concept to processes with many coloured particles^{10,11}, which are ubiquitous at hadron colliders.

Both of these questions are conveniently addressed using the path integral technique for soft gluon resummation. The essential idea of this approach is that QCD scattering processes are rewritten in terms of (first-quantised) path integrals over the trajectories of the hard emitting particles. To see what this means in more detail, consider the cartoon shown in figure 2, which shows Drell-Yan production, in which incoming quarks fuse to make a final state vector boson. If we think about this process in position space, the incoming particles can emit gluons at various places along their spacetime trajectories. We have already seen that the eikonal approximation (which gives the first set of logs in eq. (1)) corresponds to the emitted gluons having zero momentum. Then the incoming particles do not recoil, and so follow classical straight line trajectories. Beyond the eikonal approximation, each trajectory will get a small kick or wobble upon emission of a gluon. The sum over all possible wobbles that each trajectory can have is equivalent, in a well-defined sense, to a sum over possible gluon emissions of nonzero momentum. The sum over wobbles of a trajectory is nothing other than a Feynman path integral, as used in Feynman’s original formulation of quantum mechanics. Thus, it follows that there should be a

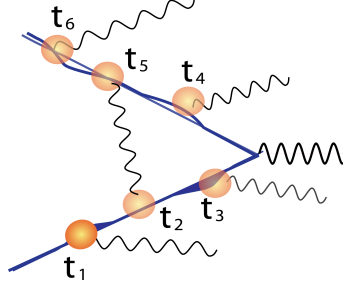


Figure 2: Spacetime depiction of Drell-Yan production, showing the trajectories of the incoming quarks.

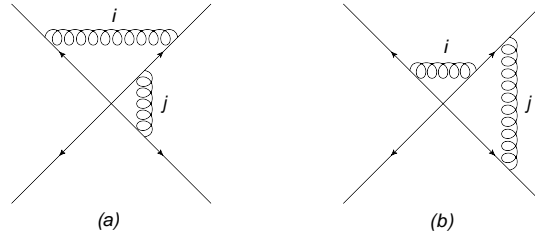


Figure 3: Example web at two loop order, for a four parton process.

description of soft gluon physics in terms of path integrals for the hard external particles, where the leading term of each path integral (the classical trajectory) gives the eikonal approximation. If one can then somehow systematically expand about the classical trajectory and keep the “first-order set of wobbles”, this gives the next-to-eikonal corrections⁸. With this approach, we have proved that the structure of NE corrections to scattering amplitudes has the generic form

$$\mathcal{A} = \mathcal{A}_0 \exp \left[\mathcal{M}^E + \mathcal{M}^{NE} \right] \times [1 + \mathcal{M}_{rem.}] + \mathcal{O}(NNE). \quad (3)$$

Here the left-hand side denotes the amplitude for a given Born interaction \mathcal{A}_0 dressed by soft and next-to-soft gluons. The first term in the exponent denotes eikonal webs^{1,2,3}, and the second term constitutes next-to-eikonal webs. Finally there is a remainder term whose interpretation is also understood⁸. The above formula has been confirmed using an explicit diagrammatic proof, and preliminary calculations in Drell-Yan production have been carried out which pave the way for resummation of next-to-eikonal effects⁹. Interestingly, the same schematic structure of next-to-eikonal corrections also holds in perturbative quantum gravity¹².

We now turn to the second open problem above, that of generalising webs (diagrams which sit in the exponent of the soft gluon amplitude) from two parton to multiparton scattering. In the two parton case of eq. (2), we saw that webs were single (irreducible) diagrams. This becomes more complicated in multiparton processes: webs are no longer irreducible, but become compound sets of diagrams, related in a particular way. Consider, for example, the two diagrams shown in figure 3, which are a two-loop soft gluon correction to a hard interaction involving four partons. Taking diagram (a), we can make a second diagram by permuting the gluons on the upper right-hand line. This gives diagram (b), and by performing the permutation again we get back the original diagram. The graphs thus form a closed set under permutations of gluon emissions. Such closed sets are argued to be the appropriate generalisation of webs to multiparton scattering¹⁰. The derivation of these results uses the *replica trick*, an elegant technique for proving exponentiation properties which is borrowed from statistical physics.

Each diagram D in a given closed set (web) has a kinematic part $\mathcal{F}(D)$ and a colour factor $C(D)$. In the normal amplitude, these are simply multiplied together. However, in the exponent of the amplitude, the colour and kinematic parts of web diagrams mix with each other. That

is, a single web contributes a term

$$\sum_{D,D'} \mathcal{F}_D R_{DD'} C_{D'} \quad (4)$$

to the soft gluon exponent, where the sum is over diagrams in the web, and $R_{DD'}$ is a *web mixing matrix* which describes how the vectors of kinematic and colour factors are entangled. The study of multiparton webs is thus entirely equivalent to the study of web mixing matrices. They are matrices of constant numbers (e.g. independent of the number of colours) that encode a huge amount of physics! An ongoing goal is to classify general properties of these matrices, and to translate these into physical results.

We already know about some interesting properties. Firstly, any row of any web mixing matrix has elements which sum to zero. Secondly, any web mixing matrix is idempotent, that is $R^2 = R$. The matrices are thus projection operators, having eigenvalues of 0 and 1 (with an appropriate degeneracy). These properties have been interpreted physically¹⁰, and the proofs use both the replica trick and known properties of combinatorics¹³. This latter point is itself interesting, as a pure mathematician could have proved these results without in fact knowing any of the underlying physics. This suggests that there are two ways of finding out more about web mixing matrices - either one may apply known physics constraints and see what this implies in web mixing matrix language, or one may study the matrices from a pure combinatorics point of view, and learn in the process about the entanglement of colour and kinematics^a!

To summarise, path integral methods prove highly powerful in analysing soft gluon physics, allowing new results to be obtained. Specifically, we have outlined the classification of next-to-eikonal corrections, and also the structure of multiparton webs. The results have application to the resummation of logarithms in cross-sections, but may also have more formal applications in elucidating the structure of scattering amplitudes in a variety of field theory contexts. Investigation of these possibilities is ongoing.

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^aNote that extra complications arise when the renormalisation of webs is considered¹¹.

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